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No. of Printed Pages : 13

Roll No. .....

2K5-AS-2

December – 2015

APPLIED MATHEMATICS–I

Time Allowed : 3 Hours

Max. Marks : 100

Note :
1. *'Part–A' may be attempted in first 6 pages of Answer-sheet.*
2. *'Part–B' in rest of the sheets of Answer-sheet.*
3. *The question paper consists of two parts, namely, Part–A & Part–B.*
4. *A candidate has to attempt both parts.*
5. *Part–A consists of two questions and Part–B consists of 5 questions.*
6. *Answer may be given in Hindi or English language.*

[Part – A]

1. Answer any 10 parts– 2×10 = 20

(i) If A is a square matrix such that $|A| = 2$, write the value of

$|AA^T|$

(ii) Write the value of x for which matrix

$$\begin{bmatrix} 6-x & 4 \\ 3+x & 1 \end{bmatrix}$$

is singular.

(iii) If

$$x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix},$$

find the value of x.

(iv) Write the value of

$$\lim_{x\to 0}\frac{\sin 3x}{x}$$

(v) Find :

$$\frac{d}{dx}\left(\log_5^{x^2}\right)$$

(vi) If $\vec{a}$ and $\vec{b}$ are diagonals of a parallelogram, then write the formula finding its area in terms of $\vec{a}$ and $\vec{b}$.

(vii) Find the angle between the vectors $\vec{a}$ and $\vec{b}$, if

$$|\vec{a}|=\sqrt{3},\ |\vec{b}|=2 \text{ and } \vec{a}.\vec{b}=\sqrt{6}$$

(viii) Write the sum of the intercepts on the coordinate axes made by the line

$$3x - 2y + 6 = 0$$

(ix) Find the eccentricity of the ellipse

$$2x^2 + 3y^2 = 6$$

(x) Write the coordinates of the centre of the circle passing through (0, 0), (a, 0) and (0, b).

(xi) Write the length of the latus-rectum of the hyperbola

$$16x^2 - 9y^2 = 144$$

(xii) Write the y-intercept of the line passing through (2, 2) and perpendicular to the line $3x + y = 3$.

(xiii) Find the distance between the lines

$$5x + 3y - 7 = 0,$$

and

$$15x + 9y + 14 = 0$$

(xiv) If $y = x|x|$,

find $\frac{dy}{dx}$

fox $x < 0$

2. Attempt any five parts : 5×4 = 20

(i) Simplify :

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

(ii) Find the value of x satisfying the equation

$$[1 \ x \ 1]\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$$

(iii) Without expanding show that

$$\begin{vmatrix} 1 & b+c & a \\ 1 & c+a & b \\ 1 & a+b & c \end{vmatrix} = 0$$

(iv) If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = 6$, find $|\vec{a} + \vec{b}|$ and $|\vec{a} - \vec{b}|$.

(v) Find the equation of a straight line which passes through the point (4, –2) and whose interception y-axis is twice that on x-axis.

(vi) If

$$y = \left(x + \sqrt{x^2 + a^2}\right)^n,$$

prove that

$$\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$$

(vii) If

$$\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80,$$

find n.

(viii) The straight line

$$\frac{x}{a} + \frac{y}{b} = 1$$

cuts the coordinate axes at A and B. Find the equation of the circle passing through O(0, 0), A and B.

[Part–B]

Attempt any three questions. 20×3 = 60

3. (a) Show that

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$$

(b) Show that the matrix

$$A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$$

satisfies the equation $x^2 - 6x + 17 = 0$. Hence, find A^{-1}.

4. (a) Find the area of the triangle whose vertices are A(3, –1, 2), B(1, –1, –3) and C(4, –3, 1).

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ, than prove that

(i) $\cos\frac{\theta}{2}=\frac{1}{2}\left|\vec{a}+\vec{b}\right|$

(ii) $\sin\frac{\theta}{2}=\frac{1}{2}\left|\vec{a}-\vec{b}\right|$

5. (a) Find the equation of the line which passes through the point (3, 4) and the sum of its intercepts on the axes is 14.

(b) Find the equation of the parabola with vertex (2, –3) and focus (0, 5).

6. (a) If

$x^m y^n = (x + y)^{m+n}$,

prove that

$\frac{dy}{dx}=\frac{y}{x}$

(b) Evaluate the following limits :

(i) $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)$

(ii) $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$

7. (a) If

$x^y = e^{x-y}$,

prove that

$\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$

(b) Find the equation of the circle circumscribing the triangle formed by the lines x + y = 6, 2x + y = 4 and x + 2y = 5.