

Applied Mathematics-I
2K5-AS-02

Time : 3 hrs.

M.M : 100

Note :—

1. Part 'A' may be attempted in first 6 pages of Answer Sheet.
भाग 'क' के सभी उत्तर, उत्तर-पुस्तिका के प्रथम छः पृष्ठों में ही करने हैं।
2. Part 'B' in rest of the Sheets of Answer Sheet.
भाग 'ख' के उत्तर, उत्तर-पुस्तिका के अगले शेष पृष्ठों में लिखिये।
3. Answers may be given in English or Hindi.
प्रश्नों के उत्तर अंग्रेजी अथवा हिन्दी में दीजिये।

PART-A

1. Attempt any 10 parts :

(10 x 2 = 20)

(a) Find the inverse of the matrix $A = \begin{bmatrix} 5 & -2 \\ 4 & -3 \end{bmatrix}$

(b) Find the product AB of the matrices $A = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 1 & -4 \end{bmatrix}$

(c) Determine the intercepts on x and y - axis for the straight line $3x+4y-12=0$

(d) Find the equation of the circle centred at $(2, -2)$ having radius 6.

(e) Find the coordinates of the vertex and focus for the parabola. $x^2+12y=0$

(f) Find the slope of a line perpendicular to the line $2x+5y+1=0$.

(g) If A is a square matrix such that $|A^T| = 8$, write the value of $|A^{-1}|$.

(h) If $\begin{bmatrix} a & 14 \\ 1 & b \end{bmatrix} + 2 \begin{bmatrix} 2 & -4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 7 & 7 \end{bmatrix}$, then find the values of a and b.

(i) If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 3\hat{k}$, determine $|\vec{a} + \vec{b}|$

(j) Find $5\vec{a} - \vec{b}$, if $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$.

(k) Find $\frac{d}{dx}(e^{ax-b})$.

(l) Find $\frac{d}{dx}(\sin^2 x)$.

(m) Evaluate $\lim_{x \rightarrow 2}(2x^2 - 4)$.

(n) Find $\frac{d}{dx}(5x^3)$.

2. Attempt any 5 part:

(5x4=20)

- (a) Convert the Cartesian co-ordinates $(-3, -\sqrt{3})$ in to the corresponding polar coordinates.
- (b) Find the equation of the line which passes through the point $(2, -4)$ and parallel to the line $8x+2y+3=0$
- (c) Evaluate the determinant of the matrix $A = \begin{bmatrix} 3 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$
- (d) If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, then find $\vec{a} \cdot \vec{b}$
- (e) If $y = \log(\operatorname{cosec} x - \cot x)$, find $\frac{dy}{dx}$.
- (f) Find the derivative of $\cos x$ from the first principles.
- (g) Compute $AB-BA$, where $A = \begin{bmatrix} 1 & 8 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix}$.
- (f) Obtain the equation of the circle having the points $(1, -2)$ and $(-1, 1)$ as the end points of a diameter.

PART- B

Attempt any 3 questions:

(3x20=60)

3. (a) Evaluate $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$

(b) If $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$, verify that $A (\operatorname{adj}A) = (\operatorname{adj}A) A = |A|I$, Where I is the identity matrix of the third order.

4. (a) A triangle is formed by the lines $x+y=3$, $2x-3y=6$ and $7x-3y+9=0$, find the co-ordinates of the centroid of the triangle.

(b) Find the equation of the circle with centre $(-1, 1)$ and touching the line $2x - y = 8$

6. (a) Find a unit vector perpendicular to each of the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - 3\hat{k}$.

(b) Find the coordinates of the centre, vertices, foci and equations of the directrices for the hyperbola $x^2 - 3y^2 + 12x + 6y + 18 = 0$.

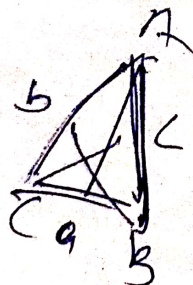
6. (a) If $y = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$, show that $\frac{dy}{dx} = \sqrt{a^2 - x^2}$

(b) If \vec{a} and \vec{b} be two - zero vectors such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, show that \vec{a} is perpendicular to \vec{b}

7. (a) Solve the following system of equations using matrices :

$$x + y + z = 3, \quad x - 2y + 3z = 4, \quad x + 4y + 9z = 6$$

(b) Show that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c)$ applying the properties of determinants.



~~g, h~~

$$x^2 + y^2 + 2gx + 2fy + c$$

$$\frac{ax + by + c}{a^2 + b^2 + c}$$

$$(h - k)$$