LTS

E-02

Applied Mathematics - I

2K5-AS-2

Roll No.: 1902[redacted]

Time : 3 Hrs. **M.M. 100**

Note :

1. *Part 'A' may be attempted in first 6 pages of Answer Sheet.*
   भाग 'क' के सभी उत्तर, उत्तर-पुस्तिका के प्रथम छः पृष्ठों में ही करने हैं।
2. *Part 'B' in rest of the Sheets of Answer Sheet.*
   भाग 'ख' के उत्तर, उत्तर-पुस्तिका के अगले शेष पृष्ठों में लिखिये।
3. *Answers may be given in English or Hindi.*
   प्रश्नों के उत्तर अंग्रेजी अथवा हिन्दी में दीजिये।

Part 'A'

भाग 'क'

1. Attempt any 10 questions. **(10x2=20)**

(i) If $\begin{bmatrix} y & -3 \\ 3 & x \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix}$

Then find the values of x and y.

(ii) Find the determinant of the matrix $A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$

(iii) Find the product of the matrices $A = \begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

(iv) Determine the intercept on x and y - axis for the straight line $3x + 4y - 12 = 0$

(v) Find the equation of circle centred at (0, 0) and radius 2.

(vi) Find the equation fo parabola whose vertex is (0, 0) and latus rectum is 4.

(vii) Find the slope of line parallel to the line $2x - 3y + 5 = 0$

(viii) Find $\vec{a} \cdot \vec{b}$ if $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{k}$.

(ix) If $\vec{a} = 2\hat{i} + 4\hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$ then find $2\vec{a} - 3\vec{b}$

(x) Find the value of a if $\vec{a} = 2\hat{i} + 3\hat{j} - a\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 2\hat{k}$ are perpendicular.

(xi) Find : $\frac{d}{dx}(x^2 + 5x)$

(xii) Find : $\frac{d}{dx}(\sin^2 x)$

(xiii) Find the derivative of $\log_e \cos x$, w.r.t.x.

(xiv) Evaluate : $\int_{-x \to -1}^{} \frac{dt}{x^2+1}$

2. Attempt any 5 questions. **(5x4=20)**

(i) Find the value of x and y from the following equation.

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

(ii) Find the equation of the circle passing through $(-1, 2)$ and radius is 3.

(iii) Find the equation of the line which passes through the point (3, 7) and parallel to the line $3x + 2y + 3 = 0$

(iv) Evaluate $\begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix}$

(v) Compute $AB + BA$ where $A = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 \\ 1 & -1 \end{bmatrix}$

(vi) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, find $|\vec{a} \times \vec{b}|$

(vii) Find the derivative of e^x w.r.t. x by first principle.

(viii) If $\vec{a} = \hat{i} - 2\hat{j}$. Find the vector parallel to $\vec{a}$ and having magnitude 10.

Part 'B'

Attempt any three questions. **(3x20=60)**

3. (a) If $y = 3e^{2x} + 2e^{2x}$, Prove that : $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

(b) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & 4 \end{bmatrix}$, Verify that $A(\text{adj}A) = (\text{adj}A)A = |A|I$

4. (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$

(b) Find the equation fo the circle passes through $(1, -6)$, $(2, 1)$ and $(5, 2)$

5. (a) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$. Find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

(b) Find centre, vertices, foci, eccentricity of the ellipse.

$x^2 + 2y^2 + 4x - 12y + 20 = 0$

6. (a) Find the area of triangle whose vertices are :

$(1, -1)$, $(-1, -1)$, $(-\sqrt{3}, \sqrt{3})$

(b) Prove that $\begin{bmatrix} a & a^2 & bc \\ b & b^2 & ac \\ c & c^2 & ab \end{bmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

by using properties of determinants

7. (a) Solve the following system of equations :

$2x + 3y + 3z = 5$

$x - 2y + 2z = -4$

$3x - y - 2z = 3$

(b) Evaluate : $\lim_{x \to 3} \left(\frac{1}{x-3} - \frac{3}{x^2 - 3x} \right)$