

Applied Mathematics-II

E-286

2K5-BS-1

Roll No.: 12121057

Time : 3 Hrs.

M.M. 100

Note :

1. Part 'A' may be attempted in first 5 pages of Answer Sheet.
भाग 'क' के सभी उत्तर, उत्तर-पुस्तिका के प्रथम पांच पृष्ठों में ही करने हैं।
2. Part 'B' in rest of the Sheets of Answer Sheet.
भाग 'ख' के उत्तर, उत्तर-पुस्तिका के अगले शेष पृष्ठों में लिखिये।
3. Answers may be given in English or Hindi.
प्रश्नों के उत्तर अंग्रेजी अथवा हिन्दी में दीजिये।

Part 'A'

Attempt any ten questions:

10x2= 20

- (i) Evaluate $\int x^2 dx$
- (ii) Find $\int (e^x + 3x) dx$
- (iii) Determine the period of $\tan(4\pi x - 3)$
- (iv) Evaluate $\int (x^2 + 5) dx$
- (v) Evaluate $\int_{-3}^{-1} |x| dx$
- (vi) If $U = \{n : n \leq 10, n \in \mathbb{N}\}$ be the universal set and $A = \{2n : n \leq 5, n \in \mathbb{N}\}$, find A'
- (vii) Write the set $\{1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \frac{6}{11}, \frac{7}{13}, \frac{8}{15}, \frac{9}{17}, \frac{10}{19}\}$ in the set builder form
- (viii) Evaluate $\int_0^1 \frac{dx}{1+x^2}$
- (ix) If $f(x)$ is an odd function defined in the interval $[-a, a]$, write the Fourier coefficients which reduce to zero in the Fourier expansion of $f(x)$.
- (x) How many ordinates are required to evaluate a definite integral by Simpson's one-third rule taking 10 equal interval?
- (xi) Obtain a_0 in the Fourier expansion of $f(x) = |\sin x|$, if $x \in [-\pi, \pi]$
- (xii) $L\{e^t\} = \underline{\hspace{2cm}}$
- (xiii) Evaluate $L^{-1}\left\{\frac{1}{s^2-4}\right\}$
- (xiv) If $L\{f(t)\} = \frac{1}{s}$ then $f(t) = \underline{\hspace{2cm}}$

2. Attempt any 5 questions.

- (i) Evaluate $\int_0^2 x(4-x^2)dx$
- (ii) Find $\int (\cos x - \sin x)dx$
- (iii) If $A = \{1, 2, 4, 5, 7\}$, $B = \{2, 3, 6, 7, 8\}$ and $C = \{3, 4, 5, 7, 9\}$ verify that $A - (B \cup C) = (A - B) \cap (A - C)$
- (iv) Evaluate $\int_0^{\pi/2} (a \sin^2 x + b \cos^2 x)dx$
- (v) Determine whether $\log(\sqrt{1+x^2} - x)$ is an even function or odd function
- (vi) Determine the Laplace transform of $e^{-3t} + t + \cos 2t$.
- (vii) Find $L^{-1} \left\{ \frac{1}{s^2 - 9} \right\}$
- (viii) Find $\int \left(\frac{9}{1+x^2} + \sin x \right) dx$

PART-B

Attempt any 3 questions:

3 X 20 = 60

3. (a) Find $\int \frac{x^2}{x^2+1} dx$
- (b) Obtain the Fourier series to represent the function $f(x) = |x|$ for $-\pi \leq x \leq \pi$.
Hence, deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$
4. (a) Evaluate $\int_0^{\pi/2} (1 + \sin x) dx$
- (b) Develop the Fourier series for the function $f(x) = \pi - x$, defined in the interval $[0, 2\pi]$
5. (a) Determine the Laplace Transform of $\sin^2(at)$ using the Laplace transform of derivative.
- (b) Evaluate $\int_0^{2a} \left(\frac{1}{x^2+a^2} + \sin ax \right) dx$
6. (a) Evaluate $\int (3e^x + 5 \sin x + \tan 2x) dx$
- (b) Using Simpson's one-third rule, find the approximate value of the integral $\int_0^1 \frac{1}{1+x} dx$ taking 6 equal intervals. Hence, obtain an approximate value of $\log_e 2$.
7. (a) There are 70 people in a group out of which 45 speak Hindi language and 33 speak English language and 10 speak neither Hindi nor English. How many speak both Hindi as well as English? How many speak only English language?
- (b) Obtain the half-range cosine series of the function $f(x) = x^2$, for $0 \leq x \leq \pi$
Hence, show that $\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$