Wireframe Model

Wireframe modeling is the process of visual presentation of a three-dimensional or physical object used in 3-D computer graphics. It is an abstract edge or skeletal representation of a real-world 3-D object using lines and curves.

Example

A wireframe is a three-dimensional model that only includes vertices and lines. It does not contain surfaces, textures, or lighting like a 3D mesh. A wireframe model is a 3D image comprised of only "wires" that represent three-dimensional shapes.

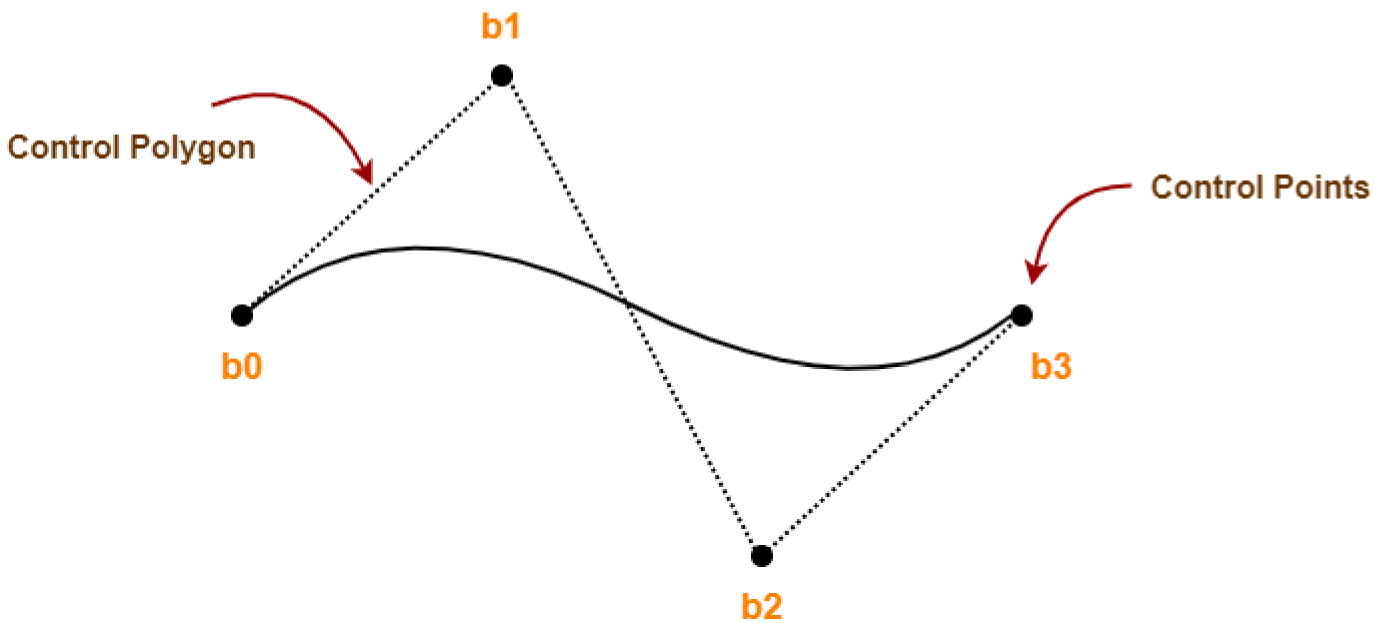
Wireframes provide the most basic representation of a three-dimensional scene or object. They are often used as the starting point in 3D modeling since they create a "frame" for 3D structures. For example, a 3D graphic designer can create a model from scratch by simply defining points (vertices) and connecting them with lines (paths). Once the shape is created, surfaces or textures can be added to make the model appear more realistic.

The lines within a wireframe connect to create polygons, such as triangles and rectangles, that together represent three-dimensional shapes. The result may be as simple as a cube or as complex as a three-dimensional scene with people and objects.

Bezier Curves

A Benzier curve is a parametric curve used in computer graphics and related fields. The curve, which is related to the Bernstein polynomial used in the 1960s for designing curves for the bodywork of Renault cars. Other uses include the design of computer fonts and animation.

The following curve is an example of a bezier curve



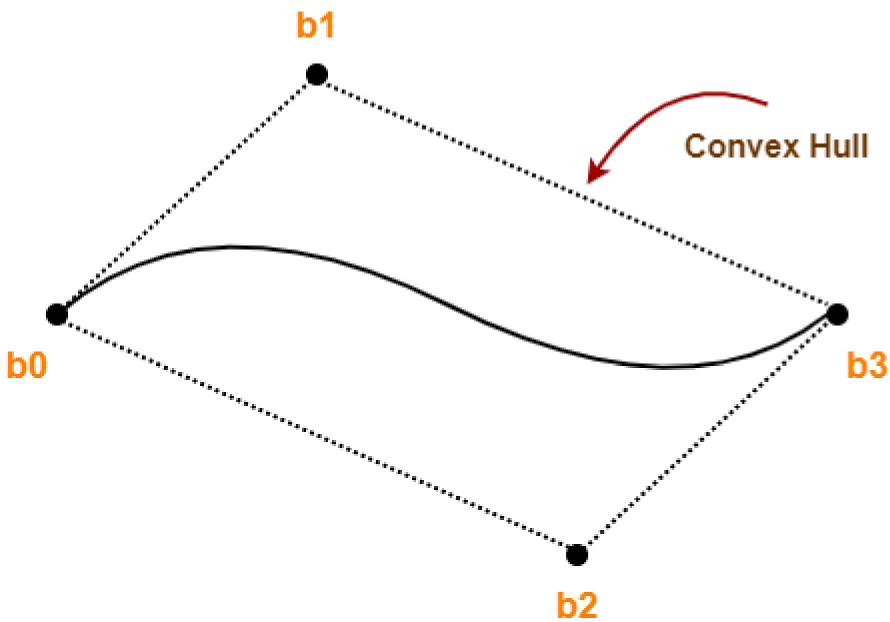
Bezier Curve Example

- This bezier curve is defined by a set of control points b_0, b_1, b_2 and b_3.
- Points b_0 and b_3 are ends of the curve.
- Points b_1 and b_2 determine the shape of the curve.

Few important properties of a bezier curve are:

<u>Property-01:</u>

Bezier curve is always contained within a polygon called as convex hull of its control points.



Bezier Curve With Convex Hull

<u>Property-02:</u>

Bezier curve generally follows the shape of its defining polygon.

- The first and last points of the curve are coincident with the first and last points of the defining polygon.

<u>Property-03:</u>

The degree of the polynomial defining the curve segment is one less than the total number of control points.

Degree = Number of Control Points – 1

Property-04:

The order of the polynomial defining the curve segment is equal to the total number of control points.

Order = Number of Control Points

Property-05:

- Bezier curve exhibits the variation diminishing property.
- It means the curve do not oscillate about any straight line more often than the defining polygon.

Bezier Curve Equation-

A bezier curve is parametrically represented by-

$$P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t)$$

Bezier Curve Equation

Here,

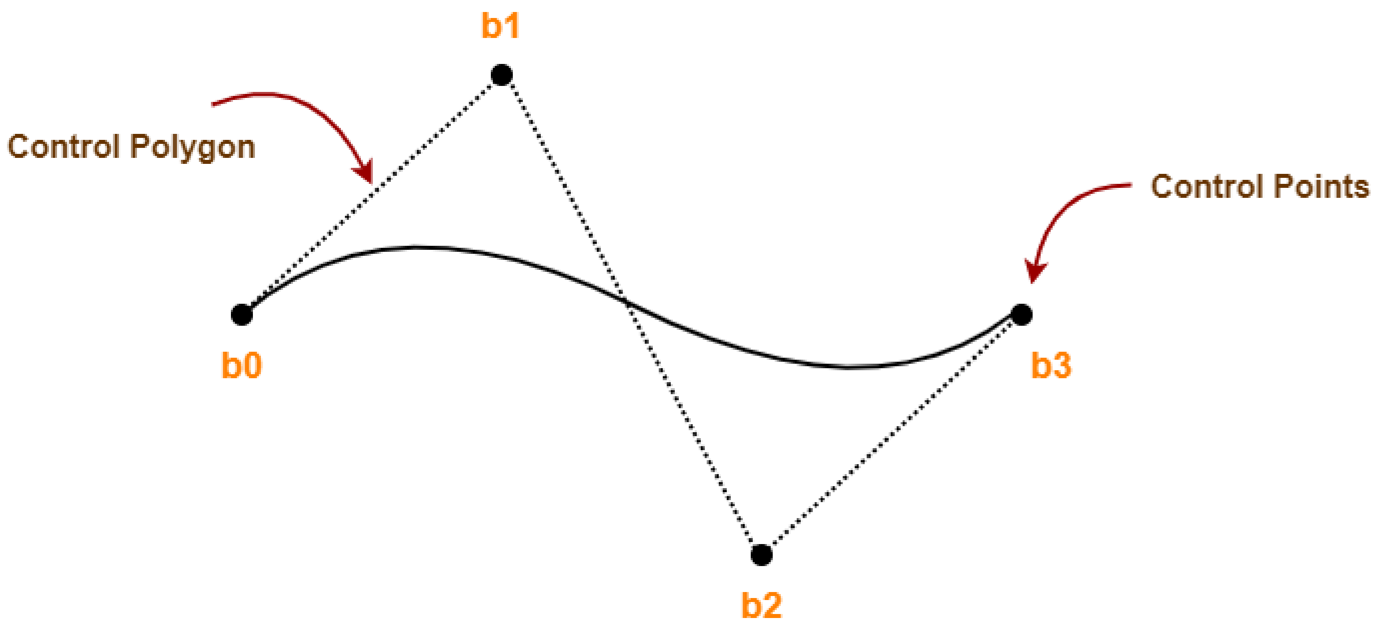
- t is any parameter where 0 <= t <= 1
- P(t) = Any point lying on the bezier curve
- B_i = i^{th} control point of the bezier curve
- n = degree of the curve
- $J_{n,i}(t)$ = Blending function = $C(n,i)t^i(1-t)^{n-i}$ where C(n,i) = n! / i!(n-i)!

Cubic Bezier Curve-

- Cubic bezier curve is a bezier curve with degree 3.
- The total number of control points in a cubic bezier curve is 4.

Example-

The following curve is an example of a cubic bezier curve-



Cubic Bezier Curve

- This curve is defined by 4 control points b_0, b_1, b_2 and b_3.
- The degree of this curve is 3.
- So, it is a cubic bezier curve.

Cubic Bezier Curve Equation-

The parametric equation of a bezier curve is-

$$P(t) = \boxed{\sum_{i=0}^{n} B_i J_{n,i}(t)}$$

$$P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t)$$

Bezier Curve Equation

Substituting n = 3 for a cubic bezier curve, we get-

$$P(t) = \sum_{i=0}^{3} B_i J_{3,i}(t)$$

Expanding the above equation, we get-

$P(t) = B_0J_{3,0}(t) + B_1J_{3,1}(t) + B_2J_{3,2}(t) + B_3J_{3,3}(t)$ ………..(1)

Now,

$$J_{3,0}(t) = \frac{3!}{0!\,(3-0)!}\, t^{0}\,(1-t)^{3-0}$$

$$J_{3,0}(t) = (1-t)^{3} \qquad \text{.............(2)}$$

$$J_{3,1}(t) = \frac{3!}{1!\,(3-1)!}\, t^{1}\,(1-t)^{3-1}$$

$$J_{3,1}(t) = 3t(1-t)^{2} \qquad \text{.............(3)}$$

$$J_{3,2}(t) = \frac{3!}{2!\,(3-2)!}\, t^{2}\,(1-t)^{3-2}$$

$$J_{3,2}(t) = 3t^{2}(1-t)^{2} \qquad \text{.............(4)}$$

$$J_{3,3}(t) = \frac{3!}{3!\,(3-3)!}\, t^{3}\,(1-t)^{3-3}$$

$$J_{3,3}(t) = t^{3} \qquad \text{.............(5)}$$

Using (2), (3), (4) and (5) in (1), we get-

$$P(t) = B_0(1-t)^3 + B_1 3t(1-t)^2 + B_2 3t^2(1-t) + B_3 t^3$$

This is the required parametric equation for a cubic bezier curve.

Applications of Bezier Curves:

1. Computer Graphics-

- Bezier curves are widely used in computer graphics to model smooth curves.
- The curve is completely contained in the convex hull of its control points.
- So, the points can be graphically displayed & used to manipulate the curve intuitively.

2. Animation-

- Bezier curves are used to outline movement in animation applications such as Adobe Flash and synfig.
- Users outline the wanted path in bezier curves.
- The application creates the needed frames for the object to move along the path.
- For 3D animation, bezier curves are often used to define 3D paths as well as 2D curves.

3. Fonts-

- True type fonts use composite bezier curves composed of quadratic bezier curves.
- Modern imaging systems like postscript, asymptote etc use composite bezier curves composed of cubic bezier curves for drawing curved shapes.

PRACTICE PROBLEMS BASED ON BEZIER CURVE IN COMPUTER GRAPHICS:

Problem-01:

Given a bezier curve with 4 control points-

$B_0[1\ 0]$, $B_1[3\ 3]$, $B_2[6\ 3]$, $B_3[8\ 1]$

Determine any 5 points lying on the curve. Also, draw a rough sketch of the curve.

Solution-

We have-

- The given curve is defined by 4 control points.
- So, the given curve is a cubic bezier curve.

The parametric equation for a cubic bezier curve is-

$$\mathbf{P(t) = B_0(1\text{-}t)^3 + B_1 3t(1\text{-}t)^2 + B_2 3t^2(1\text{-}t) + B_3 t^3}$$

Substituting the control points B_0, B_1, B_2 and B_3, we get-

$P(t) = [1\ 0](1-t)^3 + [3\ 3]3t(1-t)^2 + [6\ 3]3t^2(1-t) + [8\ 1]t^3$ ……..(1)

Now,

To get 5 points lying on the curve, assume any 5 values of t lying in the range 0 <= t <= 1.

Let 5 values of t are 0, 0.2, 0.5, 0.7, 1

For t = 0:

Substituting t=0 in (1), we get-

$P(0) = [1\ 0](1-0)^3 + [3\ 3]3(0)(1-t)^2 + [6\ 3]3(0)^2(1-0) + [8\ 1](0)^3$

$P(0) = [1\ 0] + 0 + 0 + 0$

$P(0) = [1\ 0]$

For t = 0.2:

Substituting t=0.2 in (1), we get-

$P(0.2) = [1\ 0](1-0.2)^3 + [3\ 3]3(0.2)(1-0.2)^2 + [6\ 3]3(0.2)^2(1-0.2) + [8\ 1](0.2)^3$

$P(0.2) = [1\ 0](0.8)^3 + [3\ 3]3(0.2)(0.8)^2 + [6\ 3]3(0.2)^2(0.8) + [8\ 1](0.2)^3$

$P(0.2) = [1\ 0] \times 0.512 + [3\ 3] \times 3 \times 0.2 \times 0.64 + [6\ 3] \times 3 \times 0.04 \times 0.8 + [8\ 1] \times 0.008$

$P(0.2) = [1\ 0] \times 0.512 + [3\ 3] \times 0.384 + [6\ 3] \times 0.096 + [8\ 1] \times 0.008$

$P(0.2) = [0.512\ 0] + [1.152\ 1.152] + [0.576\ 0.288] + [0.064\ 0.008]$

$P(0.2) = [2.304\ 1.448]$

For t = 0.5:

Substituting t=0.5 in (1), we get-

$P(0.5) = [1\ 0](1-0.5)^3 + [3\ 3]3(0.5)(1-0.5)^2 + [6\ 3]3(0.5)^2(1-0.5) + [8\ 1](0.5)^3$

$P(0.5) = [1\ 0](0.5)^3 + [3\ 3]3(0.5)(0.5)^2 + [6\ 3]3(0.5)^2(0.5) + [8\ 1](0.5)^3$

$P(0.5) = [1\ 0] \times 0.125 + [3\ 3] \times 3 \times 0.5 \times 0.25 + [6\ 3] \times 3 \times 0.25 \times 0.5 + [8\ 1] \times 0.125$

$P(0.5) = [1\ 0] \times 0.125 + [3\ 3] \times 0.375 + [6\ 3] \times 0.375 + [8\ 1] \times 0.125$

$P(0.5) = [0.125\ 0] + [1.125\ 1.125] + [2.25\ 1.125] + [1\ 0.125]$

$P(0.5) = [4.5\ 2.375]$

For t = 0.7:

Substituting t=0.7 in (1), we get-

$P(t) = [1\ 0](1-t)^3 + [3\ 3]3t(1-t)^2 + [6\ 3]3t^2(1-t) + [8\ 1]t^3$

$P(0.7) = [1\ 0](1-0.7)^3 + [3\ 3]3(0.7)(1-0.7)^2 + [6\ 3]3(0.7)^2(1-0.7) + [8\ 1](0.7)^3$

$P(0.7) = [1\ 0](0.3)^3 + [3\ 3]3(0.7)(0.3)^2 + [6\ 3]3(0.7)^2(0.3) + [8\ 1](0.7)^3$

$P(0.7) = [1\ 0] \times 0.027 + [3\ 3] \times 3 \times 0.7 \times 0.09 + [6\ 3] \times 3 \times 0.49 \times 0.3 + [8\ 1] \times 0.343$

P(0.7) = [1 0] x 0.027 + [3 3] x 0.189 + [6 3] x 0.441 + [8 1] x 0.343

P(0.7) = [0.027 0] + [0.567 0.567] + [2.646 1.323] + [2.744 0.343]

P(0.7) = [5.984 2.233]

For t = 1:

Substituting t=1 in (1), we get-

P(1) = [1 0](1-1)^3 + [3 3]3(1)(1-1)^2 + [6 3]3(1)^2(1-1) + [8 1](1)^3

P(1) = [1 0] x 0 + [3 3] x 3 x 1 x 0 + [6 3] x 3 x 1 x 0 + [8 1] x 1

P(1) = 0 + 0 + 0 + [8 1]

P(1) = [8 1]

Following is the required rough sketch of the curve-